Worksheet: Transformations

Transformations are mathematical actions which move a point, object or graph to a different location on the coordinate plane. In this lesson you will use Nspire technology to help you visualise translations and reflections, as well as to mathematically describe the effects of these actions.

We will use the following terminology and notation:

Source: the original point, object or graph

Image: the new point, object or graph formed by the transformation (or *mapping*) of the source.

If the source A (2, 5) is transformed to its image A´ (3, 2), we write this mapping as A → A´ or (2, 5) → (3, 2).

Activity 1: Translating a triangle

Open page 1.1 of your Nspire file. You see the source ΔABC shaded grey and formed by A (1,4), B (3,2) and C (0,–1). You also see the image ΔA´B´C´. Adjusting the sliders **h** and **v** allows you to perform separate horizontal and vertical translations on the source.

Fill in the blank or circle the correct response for the following:

1. To map ΔABC onto ΔA´B´C´, **h=____** and **v=_____**.

2. Therefore, the translation is described as shifting the source _____ units to the right / left and ____ units up / down.

3. The notation describing this translation is given by A(1,4) → A´(,); B(3,2) → B´(,) and C(0,-1)→ C´(,)

4. The notation describing this translation is in general given by $(x,y) \rightarrow (x$_____$,y$______).

Note: Be sure to reset the **h** *and* **v** *values back to zero before proceeding to next Nspire page!*

Now generalise these results for translations:

A translation may be written as $P(x,y) \rightarrow P'(x + \mathbf{h}, y + \mathbf{v})$.

5. Give a mathematical description of this translation.

Activity 2: Translating a quadratic graph

Open page 1.2 of your Nspire file. The graph you see is a parabola. We will use translations to move from source to image, and use the turning point to track any changes that occur. The equation of the source (a graph of a *quadratic* function) is $y=(x-3)^2+2$, and the equation of the image is $y=(x+2)^2-4$.

6. Use the *horizontal* and *vertical* sliders to move the source to its image. Since you needed to translate the source _______ units to the left/right and _________units up/down, the translation is: $(x,y) \rightarrow (x$ ____, y____).

Reset **h** and **v** to zero.

7. Look closely at the equations for the source and image, their turning point coordinates and the answers you wrote for (a) above. What do you notice?

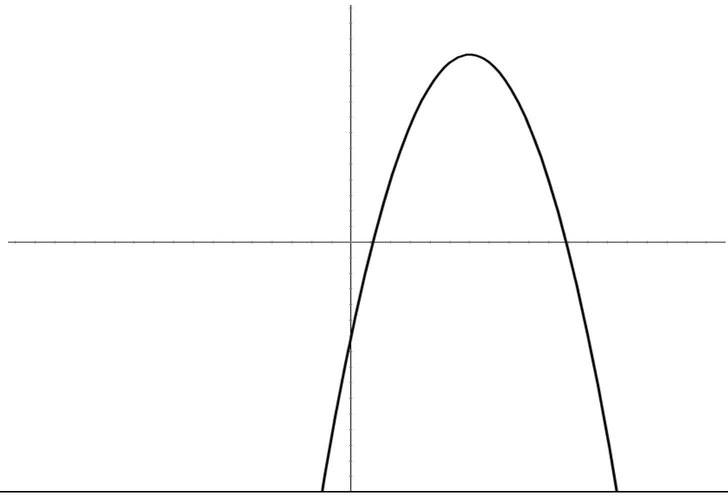
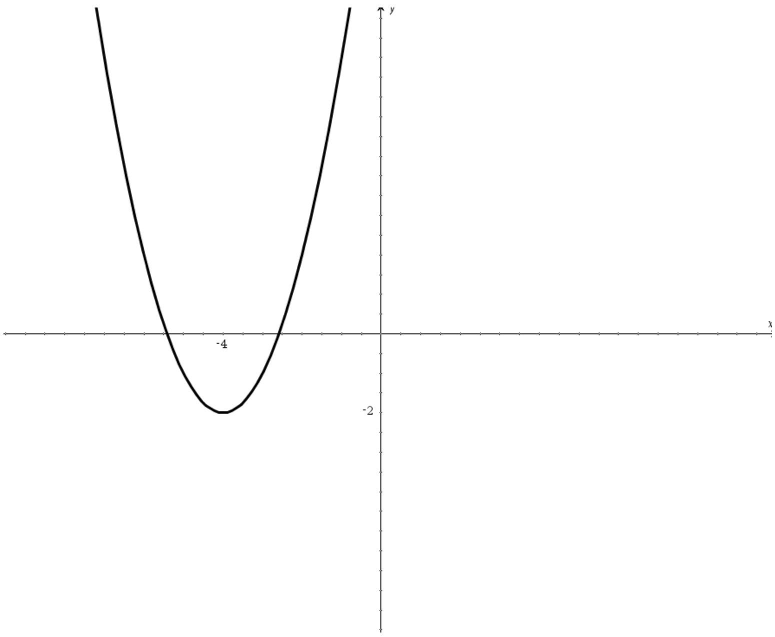
Now examine the parabolas seen on pages 1.3 and 1.4, and complete the following statements:

p1.3	p1.4
8. To map SourceParabola1 (SP1) onto ImageParabola1 (IP1) by translation, the value of **h** must be ___ and the value of **v** must be ___. 9. Write the equation of IP1 if the equation of SP1 is $y=-(x+3)^2+1$: _____________________ *Reset* **h** *and* **v**	The turning point for the given SourceParabola2 (SP2) is (-4, -1). 10. Suppose SP2 is translated using $(x,y) \rightarrow (x+5, y+4)$. The coordinates of the turning point of ImagePara bola2 (IP2) are (__, __) and its equation is ____________________. 11. Verify this by entering the right hand side (after the = sign) of your equation into the entry line at the bottom of the screen after $f8(x)$=and press **enter** enter. It should look as if it has moved _____ units to the _______ and _______ units ________.

Working out space:

Activity 3: By-hand translations

Without the assistance of technology, complete the following questions:

12. In the diagram shown, the source parabola has the equation $y = -(x-3)^2 + 6$. It is translated using the mapping $(x, y) \rightarrow (x-5, y+2)$. (a) What is the equation of the image parabola? (b) Sketch the image onto the diagram to confirm your answer.	
13. (a) Suppose the given parabola is translated so that its image is $y = (x+2)^2 + 5$. Sketch the image and describe the translation in mapping notation below: (b) Now suppose the same given parabola is the *image* of a translation described by the mapping $(x, y) \rightarrow (x-1, y-2)$. Sketch the *source* parabola using another colour, and write its equation below:	

Now check your results on a new Nspire Graphs & Geometry page in a new document, using **Menu** (menu) **6: New Document** (6).

Activity 4: Reflecting a triangle

A summary of the steps needed to create and reflect a triangle are given below:

- First make a triangle.

Press **menu** (menu) **8: Shapes** (8) 2**: Triangle** (2); move cursor to first vertex location, press enter (enter); repeat for other 2 vertices.

- Next reflect the triangle in a 'mirror'.

Press **menu** (menu) **A: Transformation** (A) 2**: Reflection** (2)

- Select the axis of reflection using the navpad (by clicking).
- Reflect the triangle across the line of reflection by selecting the triangle as before); the image will appear.

(Note that only the basic image will appear, without the source's attributes (shading, etc) and labels)

Now open page 1.5 of your Nspire file.

14. Reflect ΔJKL [with J(2,4), K(3,2), and L(6,1)] across the *x*-axis, forming ΔJ´K´L´.

15. What are the coordinates of L´? (____,____).

16. Describe in words what has happened to the coordinates of all points in the source triangle:

____.

17. Reflect ΔJKL across the *y*-axis, forming ΔJ´´K´´L´´.

18. Describe this reflection, or mapping, in symbols: $(x,y) \rightarrow$ (____,____).

19. You now have triangles in quadrants I, II and IV. Using each of these triangles, in turn, as the source, describe in words the reflection/s required to produce the image ΔJ´´´K´´´L´´´ in Quadrant III.

- Source ΔJKL maps to image ΔJ´´´K´´´L´´´ by ________________________________
- Source ΔJ´K´L´ maps to image ΔJ´´´K´´´L´´´ by ______________________________
- Source ΔJ´´K´´L´´ maps to image ΔJ´´´K´´´L´´´ by ____________________________

Activity 5: Reflecting quadratic graphs

Now open page 1.6 of your Nspire file. On this page reflections across the axes may be done on your handheld device using a slider. Note that the sliders have been set to values of only $^{-}1$ or $^{+}1$.

20. Manipulate the sliders, then describe in words the mathematical effects on SP3 of

- **m** being equal to $^{-}1$:____
- **n** being equal to $^{-}1$:____

As seen in Activity 2, the coordinates of key points can be used to understand the algebra behind these transformations. SP3 has the equation $y=(x-3)^2+2$, its turning point is T(3,2), and another point on the parabola is D(5,6).

Recall the work done in Activity 4 and manipulate the sliders to answer the questions below:

21. When **m** = $^{-}1$ and **n** = $^{+}1$ the image coordinates are T´(___, ___) and D´(___, ___); the reflection is mapped using $(x,y) \rightarrow$ (____,____), and the equation is ________________.

22. When **m** = $^{+}1$ and **n** = $^{-}1$ the image coordinates are T´´(___, ___) and D´´(___, ___); the reflection is mapped using $(x,y) \rightarrow$ (____,____), and the equation is ________________.

23. When **m** = $^{-}1$ and **n** = $^{-}1$ the image coordinates are T´´´(___, ___) and D´´´(___, ___); the reflection is mapped using $(x,y) \rightarrow$ (____,____), and the equation is ________________.

Using your growing knowledge of transformations , look at the graph seen on Nspire page 1.7 and answer the questions posed below:

SP4 is described by the rule $y=-(x-2)^2+1$.

Karen and Elizabeth are asked to transform SP4 using different methods. Karen decides to use a translation, while Elizabeth uses a reflection. When they hand in their work, their teacher notices that their image parabolas are *identical*.

24. Describe what happened in words and symbols.

25. Is this the *only* way this could have happened? Explain.

Activity 6: Extension - Compositions of transformations

More complex changes occur if we blend say, reflections and translations into a *composition* of transformations. Consider the case of a point P(2, 5) first being reflected across the *y*-axis, and then that temporary image being translated 4 units to the right and 3 units down. Symbolically this is:

$$(2,5) \xrightarrow{\text{reflection in } y\text{-axis}} (\_\_,\ \_\_) \xrightarrow{\text{translation } 4\rightarrow 3\downarrow} (\_\_,\ \_\_)$$

What if the *order* of transformations is reversed?

$$(2,5) \xrightarrow{\text{translation } 4\rightarrow 3\downarrow} (\_\_,\ \_\_) \xrightarrow{\text{reflection in } y\text{-axis}} (\_\_,\ \_\_)$$

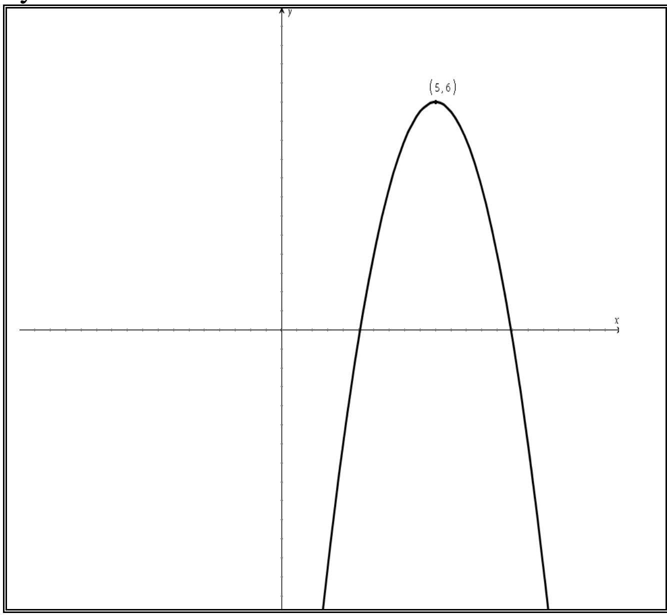
Or if, say the translation was "interrupted" by the reflection?

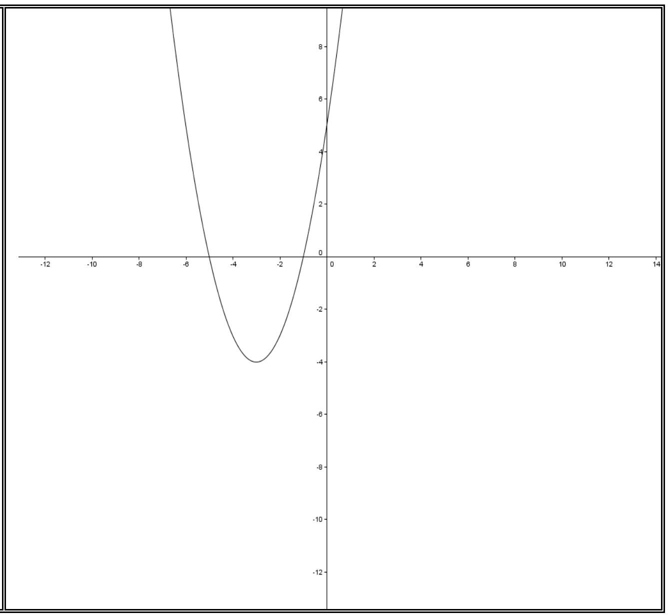
$$(2,5) \xrightarrow{\text{translation } 4\rightarrow} (\_\_,\ \_\_) \xrightarrow{\text{reflection in } y\text{-axis}} (\_\_,\ \_\_) \xrightarrow{\text{translation } 3\downarrow} (\_\_,\ \_\_)$$

Or, $$(2,5) \xrightarrow{\text{translation } 3\downarrow} (\_\_,\ \_\_) \xrightarrow{\text{reflection in } y\text{-axis}} (\_\_,\ \_\_) \xrightarrow{\text{translation } 4\rightarrow} (\_\_,\ \_\_)$$

26. What did you notice?

Now look at the graphs of the quadratic functions $y = -(x-5)^2 + 6$ and $y = (x+3)^2 - 4$. Use your knowledge of transformations (without the assistance of technology) to comment on whether *order* is important for the compositions of transformations below. Open a new Nspire document to check your answers.





27. Reflection in the *x*-axis   Translation 5 units to the left   Translation 1 unit up.

28. Reflection in the *y*-axis   Reflection in the *x*-axis   Translation 3 units to the right

29. Reflection in the *y*-axis   Translation 1 unit to the right   Translation 6 units down