LESSON PLAN -Transformations

Visualisations using Nspire CAS (Year 10)

Grade level: Year 10

Activity time: 80 minutes

Aims: The students will

- Use technology to visualize the effects of translations and reflections in the coordinate plane on geometric shapes as well as graphs of quadratic functions
- Describe these effects in words (using correct mathematical terminology) and symbols (using mapping notation and algebraic equations)
- Recognise transformations used when given equations for both source (original) and image graphs
- Explore compositions of transformations and investigate commutativity within these compositions.

Assumed concept knowledge: Horizontal and vertical translations; reflections in the *x*- and *y*-axes; graphs of quadratic functions; conventional quadrant numbering system.

Assumed Nspire knowledge (students):

- Downloading Nspire files from teacher handheld device
- Moving between pages of an Nspire multi-page document
- *Graphs&Geometry application*: selecting a shape, graph or an axis using NavPad; "grabbing" a point on a slider and moving it using NavPad; entering a function.

Additional assumed Nspire knowledge (teachers):

- Downloading an Nspire file to student handheld devices
- Using Nspire CAS Teacher Edition computer software (handheld + emulator view) with data projector

BACKGROUND

Summary of Lessons: Students are presented with a worksheet supported by a series of Nspire pages. They are directed to perform various manipulations of the objects contained on those pages and record their findings in both words and symbols.

Activity Number	Activity Name	Overview of tasks
1	**TRANSLATING A TRIANGLE**	Teacher and students use Nspire p1.1 to practise translation of a source triangle to its image using sliders, and write notation describing this. Students generalise for any translation writing worded and symbolic summaries.
2	**TRANSLATING A QUADRATIC GRAPH**	Teacher and students use Nspire p1.2 to practise translation of a parabola to its image using sliders, and compare the given equations with the details of the vertical and horizontal movements. Students work on similar tasks in p1.3-4, writing worded summaries and testing predicted equations on Nspire.
3	**BY-HAND TRANSLATIONS**	Students apply knowledge to two questions with given graphs and some translation information. They provide missing graphs, mapping notation and equations of source or image parabolas as required. Technology is not used.
4	**REFLECTING A TRIANGLE**	Teacher and students use Nspire p1.5 to practise reflection of a triangle across each axis using the reflection tool.
5	**REFLECTING QUADRATIC GRAPHS**	Teacher and students use Nspire p1.6 to practise reflection of a quadratic graph across each axis, and then both axes, using sliders. Students work on similar tasks in p1.7, describing the effects of reflection/s in both worded and symbolic forms.
6	**EXTENSION – COMPOSITION OF TRANSFORMATIONS**	Students use their acquired knowledge to transform a point – first using a reflection followed by a translation, then reversing these two procedures. They then apply any observations made to compositions involving quadratic graphs whose equations are given. They may insert an additional Nspire Graphs & Geometry page to verify their results, though this may not be necessary.

Technical notes: The diagrams used on Nspire will already be drawn. Resetting slider values to 0 before proceeding to the next page is important. On p1.6, the entry line at the bottom of the screen must read "$f8(x)=$" before the function rule is keyed in by students; all other function values $f1(x)$ to $f23(x)$ are used in supporting other graphs. The spreadsheet pages 1.8 to 1.11 should not be altered.

BACKGROUND

Nspire files

- If teachers are going to use the "TI-Smartview™ Emulator - Handheld+Sidescreen" *or* "Handheld" view when presenting, the only file required is StudentTrans.tns. (Recommended)
- If teachers are going to use the "Normal" view when presenting, both StudentTrans.tns and TeacherTrans.tns are required. The content of these files is identical, but the layout is messy if running the student version on the computer in Normal view.

Staff can download these files to their laptops, and then onto students' handhelds using Nspire Computer Link software. It is be assumed that Nspire CAS Teacher Edition version 1.7 or later is used by teachers, and Nspire CAS Version 1.7 from 2009-05-19 or later is used by students (Home, then 8:System Info, then 4:About). Enough time should be allowed for all students to have the file ready to use before the lesson starts.

Guide to Contents of Nspire file

Nspire Page No.	**Contents** (Note: *Graphs & Geometry application used unless otherwise stated*)	**Activity**
1.1	2 triangles and 2 sliders – teacher demonstrating translation	**1**
1.2	2 parabolas and 2 sliders – teacher demonstrating translation	**2**
1.3	2 parabolas and 2 sliders – students practising translation	
1.4	1 parabola – students enter function in entry line after "*f*8(*x*)=" confirming translation	
1.5	1 triangle – teacher demonstrating reflection by use of reflection tool	**4**
1.6	1 parabola and 2 sliders – teacher demonstrating reflection	**5**
1.7	2 parabolas – students examine graphs' relative positions only (no manipulation or entry)	
1.8-11	Lists & Spreadsheets – supporting data for movable graphs and shapes (Do not alter)	

Assessment and evaluation: Students could be asked to submit the worksheet booklet, including Activity 6 for which no Nspire support pages have been provided, for teacher assessment of completion.

Further Extensions: Able students may be interested to develop similar Nspire pages using other geometric shapes (lines, circles, regular or irregular polygons) using sliders and the scatterplot mode in the Graphs & Geometry and Lists & Spreadsheet applications. They could also use the translations tool, and explore the use of the rotations tool for simple shapes. Some may also be set the task of examining and then reporting on how the underlying data structure found in pages 1.8-1.11 of the Lists & Spreadsheet application supports the diagrams found in pages 1.1-7.

Technology requirements for lesson: Student Nspire handhelds to have file StudentTrans.tns downloaded prior to lessons. Teacher requires a data projector and a laptop with either StudentTrans.tns or TeacherTrans.tns running, depending on choice of mode for viewing (see "Nspire files", top p3 of this document.) Depending on preference, teacher may also run Word to project StdntWkshtTrans.doc on screen/e-whiteboard.

Other materials: Each student must have 2 different coloured pens/pencils and the "Transformations" student worksheet (from file StdntWkshtTrans.doc). Teacher needs whiteboard markers for informal comments/clarification.

Lesson (1)

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
10 min	Page 1.1	*Ensure all students have a worksheet and pen/pencil, as well as the file* StudentTrans.tns *open on their Nspire handheld. Read aloud and clarify notation and terminology information at top of worksheet page 1.* **1. TRANSLATING A TRIANGLE** • Project Page 1.1 of Nspire file on screen; demonstrating use of sliders to translate triangle ABC. Students to do same on handheld. • Reset both **h** and **v** sliders to 0, instruct students to do same. • Instruct students to complete generalised worksheet questions. *Worksheet Answers*: (Q1) -2;1 (Q2) 2 left; 1 up (Q3) (-1,5);(1,3);(-2,0) (Q4) +h,+v (Q5) (Generalisation) For the translation described as $(x,y)\rightarrow(x+h,y+v)$, the source is translated h units horizontally (left if $h<0$, right if $h>0$) and v units vertically (down if $v<0$, up if $v>0$).	Read top of p1 of Student Worksheet Open Nspire p1.1 Complete Steps 1-4 of Activity 1 Complete generalisation.	Use NavPad to move arrow on screen to sliders, where arrow will become an open hand; press **ctrl** (ctrl) **click** (·) to close hand; use NavPad to tap left and right (not hold it down) to move slider gradually; press **esc** (esc) to release hand. Resetting both sliders to zero before commencing next activity is essential.

Lesson (2)

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
10 minutes	Pages 1.2,3,4	**2. TRANSLATING A QUADRATIC GRAPH** • Project Page 1.2 of Nspire file on screen. Read through information on Worksheet page 2 regarding the parabola, assuring students this is all they need to know about such graphs for the time being. • Demonstrate use of sliders to translate source parabola to image. Students to do same on handheld. • Discuss student answers to (a) and (b) • Reset both sliders to 0, instruct students to do same. • Project Nspire pages 1.3 and 1.4 Instruct students to complete related worksheet questions, resetting **h** and **v** to 0. *Worksheet Answers*: (Q6) 5; left; 6; down; -5, -6. (Q7) When 5 is added to (or -5 subtracted from) x-3, it becomes x+2; when 6 is subtracted from 2, it becomes -4. (Q8) 3; 2 (Q9) $y = -(x+0)^2+3$, or $y = -x^2+3$ (Q10) (1,3); $y = -(x-1)^2+3$ (Q11) 5; right; 4; up.	Read top of p2 of Student Worksheet Open Nspire p1.2; complete Q (a) and (b) of Activity 2 as directed by teacher Open Nspire page 1.3 and manipulate sliders to answer worksheet questions. Open Nspire page 1.4 and use acquire knowledge to complete worksheet questions.	When students enter function on Nspire p1.4. they must do this across from the symbol $f8(x)=$; may need to press **tab** (tab) key and **enter** (≈ enter) to show this line. They must not delete any other existing functions.

Lesson (3)

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
10 minutes	Pages 1.7,8,9	**3. BY-HAND TRANSLATIONS** • This activity is technology-free. Using technology to check answers at discretion of teacher. • Students complete Questions 12 and 13. They are required to apply their knowledge of translations and quadratic graphs to questions where the given information varies. • They are required to provide image and source graphs, and to describe translations using the mapping notation as well as equations. *Worksheet Answers*: (Q12a) $y = -(x+2)^2 + 8$ (Q12b) Parabola with tp (-2,8) (Q13a) Parabola with tp (-2,5); $(x, y) \to (x+2, y+7)$; (Q13b) Parabola (-5,0); $y = (x+5)^2$	Read through questions 12 and 13 seen on p3 of Student Worksheet; work through answers under teacher guidance.	If permission is given to use technology to check by-hand work, it is suggested that students open a new document so that references to page numbering in this file are preserved.

Lesson (4)

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
10 minutes	Page 1.5	**4. REFLECTING A TRIANGLE** • Read and discuss technique for using reflections tool as seen on top of Worksheet p4. • Project Page 1.5 of Nspire file on screen. Guide students through correct answers and observations regarding reflections of triangles. • Discuss student answers to Q15, 16, 18 and 19. *Worksheet Answers*: (Q15) (6, -1) (Q16) Their x-coordinates stay the same, but their y-coordinates change sign. (Q18) $(-x, y)$ (Q19) reflection across both axes; reflection across y-axis; reflection across x-axis.	Locate reflections tool in transformations menu as shown on top of Worksheet p4. Open Nspire p1.5 Read through questions 14-19 on Student Worksheet; work through answers under teacher guidance.	In using the reflections tool (see top of Worksheet p4), the order in which items are selected is important; the line of reflection is chosen before the object to be reflected.

Lesson (5)

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
10 minutes	Pages 1.6,7	**5. REFLECTING QUADRATIC GRAPHS** • Project Page 1.6 of Nspire file on screen. • Demonstrate use of sliders to reflect SourceParabola3 to its image. Students to do same on handheld. • Discuss student answers to Q20-23 on Worksheet p5. • Project Nspire page 1.7. Instruct students to complete related worksheet questions 24 and 25 regarding SP4 – this may require a prompt to consider lines of reflection other than the axes. *Worksheet Answers*: (Q20) reflection across x-axis; reflection across y-axis (Q21) (3, -2); (5, -6); $(x, -y)$; $y = -(x-3)^2-2$ (Q22) (-3, 2);(-5, 6); $(-x, y)$; $y = (x+3)^2+2$ (Q23) (-3, -2); (-5, -6); $(-x, -y)$; $y = -(x+3)^2-2$ (Q24) Orientation is unchanged in a translation, so the image must be an inverted parabola. Hence, the Elizabeth's reflection is across the y-axis, using $(x, y) \to (-x, y)$. Karen's translation is horizontally 4 units to the left, using $(x, y) \to (x-4, y+0)$. The resulting image parabola has the equation $y = -(x+4)^2-1$. (Q25) The reflection could be across *any* vertical line, and thus there is an infinite number of solutions; *eg* if the line of reflection was $x=7$, the translation would be described as $(x, y) \to (14-x, y+0)$ and the equation of the image would be $y= -(x-12)^2-1$, etc.	Open Nspire p1.6 Read through questions 20-23 on Student Worksheet p5; work through answers under teacher guidance. Open Nspire page 1.7, using past work to answer worksheet questions 24 and 25.	These sliders **m** and **n** will only change value when moved close to 1 or -1; thus, the tapping suggested for other sliders may be better replaced here by careful dragging.

Lesson (6)

Time	Nspire CAS computer file page reference	Teacher activity	Student activity	Technology tips
Homework or Next class	n/a	*Depending on time this might be set entirely as an out-of-class activity for more capable students. It may also be started in class with all students, using varying degrees of teacher input as warranted.* **6. EXTENSION – COMPOSITION OF TRANSFORMATIONS** The idea here is to note that the order of the transformations can make a difference in the final image produced. *Worksheet Answers*: Original order [Refl Trans]: (-2, 5); (2, 2) Reversed order [Trans Refl]: (6, 2); (-6, 2) Interrupted order 1 [Trans Refl Trans]: (6, 5); (-6, 5); (-6, 2). Interrupted order 2 [Trans Refl Trans]: (2, 2); (-2, 2); (2, 2). (Q26) The image was influenced by the initial choice of transformation; "interruption" of the translation did not influence the image when first translation was perpendicular to line of reflection, but did influence image if parallel.. (Q27) $y = -(x-5)^2 + 6$: as stated, image $y = x^2 - 5$ reversed, image $y = x^2 - 7$ $\therefore$ order matters. $y = (x+3)^2 - 4$: as stated, image is $y = -(x+8)^2+5$; reversed, image is $y = -(x+8)^2+3$ $\therefore$ order matters. (Q28) and (Q29) similar results.	Complete Worksheet Activity 6. Redrawing image graphs on given axes important; students may wish to open a new document to verify graph work. Complete extensions as indicated by teacher	Use Menu (menu) 6: New Document (6).and remember to save original document when prompted.